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UNSTEADY STATES IN NON-EQUILIBRIUM TWO-PHASE SEEPAGE†

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Non-equilibrium effects in two-phase seepage, associated with a change in the rheological properties of the microemulsions formed during the motion of a water–oil mixture in a porous medium, are considered. It is shown, on the basis of an asymptotic and numerical analysis of the mode proposed, that oscillations in the pressure drop which occur during two-phase seepage can be the result of the combined effect of non-linear rheological properties and the lag in the processes involving the restructuring of microemulsion systems.

The dependences of the pressure drop on time, obtained during laboratory investigations of samples of a porous medium, are significantly different from the theoretical curves calculated using the Muskat–Leverett models of two-phase seepage. When a two-phase mixture flows through a porous medium, oscillations in the pressure drop are observed for which there is no satisfactory explanation. It is shown below that these oscillations are associated with non-equilibrium effects. Non-equilibrium effects due to processes involving the redistribution of the phases in pores have been considered previously in [1, 2]. The effect of the microemulsion state of the part of the masses in the seepage flow on the phase permeabilities has been analysed in [3]. In this paper, it is proposed to take account of the non-equilibrium effects associated with a charge in the rheological properties in microemulsified media. Here, the two-phase fluid is in the form of a microemulsion; the particles of which possess viscoelastic properties. During the motion through the porous channels, which is accompanied by deformation of the particles, there is change in the seepage resistance of the flow due to restructuring of the microemulsion with a relaxation time which is characteristic for the given system.

Taking account of these concepts, a mathematical model of the process in which oil is displaced from a sample of a porous medium by water is then considered.

1. EQUATIONS OF UNSTEADY SEEPAGE

Laboratory experiments show that, when the pressure gradient is increased, the resistance to seepage in the flow of a microemulsion is reduced and increases when the pressure gradient is decreased. In this sense, the rheological behaviour of a microemulsion is of a non-Newtonian nature (the effective viscosity decreases with increasing applied stress). The reasons for this behaviour are associated with structural rearrangements in microemulsions. Several possible details of this process have been proposed have been proposed in [3]. However, not all its special features have been fully investigated. We will therefore confine ourselves to a phenomenological approach to the description of phenomena of this kind and the specification of functions of the relative phase permeabilities (RPP) of water and oil, taking account of their non-equilibrium nature.

Using well-known approaches to the simulation of non-equilibrium flows in porous media [4], we will write the kinetic equations for the non-equilibrium RPP of water (the displacing phase) $n_1(s)$ and oil (the displaced phase) $n_2(s)$ in the form

$$n_i + \tau_1 \frac{\partial n_i}{\partial t} = k_i(s)\psi(q), \quad i = 1, 2; \quad q = |\partial p / \partial x| \tag{1.1}$$

where τ_1 is the restructuring time, $k_1(s)$, $k_2(s)$ are the equilibrium relative phase permeabilities of water and oil, respectively, s is the saturation of the displacing phase and p is the pressure. The introduction

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of the function $\psi(q)$ enables one to describe the non-Newtonian properties of the water–oil microemulsion

$$\psi = \begin{cases} 1 - \exp(-bq), & \partial q/\partial t > 0 \\ 0{,}01[\exp(bq) - 1], & \partial q/\partial t < 0 \end{cases} \tag{1.2}$$

The values of this function for the same absolute values of the pressure gradient, determined for a decreasing and an increasing pressure gradient, are not the same, that is, the phase permeabilities exhibit hysteresis.

The equations of motion and continuity for the displacing and displaced phases are written in the form

$$\mathbf{V}_1 = -\frac{kn_1}{\mu_1}\operatorname{grad} p, \quad \mathbf{V}_2 + \tau_2\frac{\partial \mathbf{V}_2}{\partial t} = -\frac{kn_2}{\mu_2}\operatorname{grad} p \tag{1.3}$$

$$s\beta_1^*\frac{\partial p}{\partial t} + m\frac{\partial s}{\partial t} + \operatorname{div}\mathbf{V}_1 = 0 \quad (1-s)\beta_2^*\frac{\partial p}{\partial t} - m\frac{\partial s}{\partial t} + \operatorname{div}\mathbf{V}_2 = 0 \tag{1.4}$$

where μ_1, μ_2 are the viscosities of the displacing and the displaced phases, $\mathbf{V}_1$, $\mathbf{V}_2$ are the seepage rates of the phases, τ_2 is the relaxation time, β_1^*, β_2^* are the coefficients of elastic compliance of the phases, k is the absolute permeability of the porous medium and m is the porosity.

For generality, the equation of motion of the displaced phase is taken in the relaxation form [4].

Equations (1.1)–(1.4) are used to describe the process of the immiscible displacement from a sample of a porous medium where a constant flow rate of the displacing fluid is maintained in the inlet cross-section and the initial pressure in the model of the porous medium is maintained at the outlet. The initial and boundary conditions, which close the system of equations, therefore have the form

$$\begin{gathered} n_1(x,0) = 0, \quad n_2(x,0) = F_2 - \text{const}, \quad s(x,0) = s_c, \quad p(x,0) = p_0 \\ s(0,t) = s_T, \quad V(0,t) = V_0, \quad p(L,t) = p_0 \end{gathered} \tag{1.5}$$

where s_c and s_T are the initial and final (limiting) saturation of the displacing phase, F_2 is the value of the RPP of the displacing fluid when $s = s_c$, p_0 is the initial pressure in the model of the stratum, $V = V_1 + V_2$ is the overall seepage rate, V_0 is the seepage rate in the inlet cross-section of the model of a porous medium and L is the length of the model.

We introduce the dimensionless variables

$$\begin{gathered} \bar{x} = \frac{x}{L}, \quad \bar{p} = \frac{p}{p_0}, \quad \bar{q} = \frac{qL}{p_0}, \quad \bar{V}_i = \frac{V_i}{V_0}, \quad \bar{V} = \frac{V}{V_0} \\ \bar{t} = \frac{V_0 t}{mL}, \quad \bar{\tau}_i = \frac{V_0\tau_i}{mL}, \quad i = 1,2 \end{gathered}$$

The bars are henceforth omitted. After some reduction, the continuity equations (1.4) are written in dimensionless form as

$$\begin{gathered} \lambda\frac{\partial p}{\partial t} - a\frac{\partial}{\partial x}\left[(n_1 + \mu_0 n_2)\frac{\partial p}{\partial x}\right] - \tau_2\frac{\partial}{\partial x}\frac{\partial V_2}{\partial t} = 0 \\ \lambda_1 s\frac{\partial p}{\partial t} + \frac{\partial s}{\partial t} + \frac{\partial}{\partial x}(FV) + \tau_2\frac{\partial}{\partial x}F\frac{\partial V_2}{\partial t} = 0 \\ \lambda = \frac{\beta^* p_0}{m}, \quad a = \frac{kp_0}{V_0\mu_1 L}, \quad \lambda_1 = \frac{\beta_1^* p_0}{m}, \quad F = \frac{n_1}{n_1 + \mu_0 n_2}, \quad \mu_0 = \frac{\mu_1}{\mu_2} \\ V = -\int_0^x \lambda\frac{\partial p}{\partial t}dx + 1, \quad \beta^* = \frac{p_0}{m}\left[s\beta_1^* + (1-s)\beta_2^*\right] \end{gathered} \tag{1.6}$$

The system of equations (1.6) is decomposed with respect to the physical parameters and this enables one successively to calculate the pressure p using the first equation, and using this pressure, to calculate the saturation s from the second equation.

Correspondingly, Eqs (1.1) and (1.3) and the initial and boundary conditions (1.5) are reduced to dimensionless form.

2. ASYMPTOTIC ANALYSIS

The complete mathematical model in the form of Eqs (1.1)–(1.4) with the corresponding initial and boundary conditions is practically inaccessible for analytical investigation due to its complexity. We will therefore confine ourselves to an asymptotic analysis of the ordinary differential equation for the pressure gradient q. Equations (1.6) are simplified when the effects of inertia and the compressibility of the phases are neglected (that is, when $\tau_2 = 0$, $\beta_1^* = 0$, $\beta_2^* = 0$, $V = 1$).

Integrating the first of these and transforming using Eqs (1.1), instead of (1.6), we obtain

$$\tau_1 \frac{\partial q}{\partial t} - q + q^2\varphi(s)\psi(q) = 0, \quad \frac{\partial s}{\partial t} + \frac{\partial F}{\partial x} = 0$$
$$\varphi(s) = a(k_1(s) + \mu_0 k_2(s)) \tag{2.1}$$

As the final result, we arrive at the system of equations (1.1), (2.1) with the initial and boundary conditions

$$q(x,0) = q_0, \quad s(x,0) = s_c, \quad n_1(x,0) = 0, \quad n_2(x,0) = F_2, \quad s(0,t) = s_T$$

It is assumed, in determining the initial condition for the pressure gradient q, that the pressure redistribution time due to the compressibility of the fluids is negligibly small compared with the displacement time. It follows from this that the unsteady pressure redistribution processes due to elasticity effects are complete at the beginning of the displacement process. It can therefore be assumed on the "slow" time scale t that, up to the beginning of the displacement process, the pressure gradient attains its initial value q_0 at all points of the flow domain.

Further simplifications are achieved due to the smallness of the parameter τ_1 which is of the order of 10^{-3}–10^{-2} in the processes under consideration.

It should be noted that it is impossible here simply to neglect terms with the small parameter τ_1 since the perturbations are of a singular character and, moreover, it is precisely the terms $\tau_1 \partial n_i/\partial t$ which are responsible for the rapid oscillations which arise in the system. The simplification is possible here due to the separation of the fast and slow processes. On introducing a "fast" time $T = t/\tau_1$, within the framework of the method of two-scale expansions, we obtain

$$\frac{\partial q}{\partial T} - q + \tau_1 \frac{\partial q}{\partial t} = -q^2\varphi(s)\psi(q), \quad \frac{\partial s}{\partial T} + \tau_1\left[\frac{\partial s}{\partial t} + F'(n_1, n_2)\frac{\partial s}{\partial x}\right] = 0 \tag{2.2}$$
$$\frac{\partial n_i}{\partial T} + n_i + \tau_1 \frac{\partial n_i}{\partial t} = k_i(s)\psi(q), \quad i = 1,2 \quad \left(F' = \frac{dF}{ds}\right)$$

The asymptotic with respect to a small parameter q, s, n_i is constructed for the functions τ_1 in the form

$$q = q^0(x,t,T) + \tau_1 q^1(x,t,T) + O(\tau_1^2), \quad s = s^0(x,t,T) + \tau_1 s^1(x,t,T) + O(\tau_1^2) \tag{2.3}$$
$$n_i = n_i^0(x,t,T) + \tau_1 n_i^1(x,t,T) + O(\tau_1^2), \quad i = 1,2$$

It follows from Eqs (2.2) that the function $s^0 = s^0(x, t)$ is independent of the first time T. The inhomogeneous equation

$$\frac{\partial s^1}{\partial T} + \frac{\partial s^0}{dt} + F'(n_1^0, n_2^0)\frac{\partial s^0}{\partial x} = 0$$

is obtained for the correction s^1 in the following step.

Assuming that the dependence on the fast time T is periodic, from this equation, after integration over the period of the oscillations T^*, we obtain the equation for $s^0(x, t)$ in the slow variables

$$\frac{\partial s^0}{\partial t}+\frac{\partial s^0}{\partial x}\int_0^{T^*} F'(n_1^0,n_2^0)dT=0 \qquad (2.4)$$

Hence, the system of equations (2.2), which has been mainly supplemented by Eq. (2.4), turns out to be a closed system with respect to the main terms of the asymptotic: q^0, s^0, n_1^0, n_2^0.

Assuming that the function of the slow variables $s^0(x, t)$ is known, the equation for q^0 can be considered as an ordinary non-linear differential equation. The analysis of the steady-state solutions with respect to T and their stability reduces to an analysis of the equations for q^0 since s^0 is independent of time T. The steady-state solutions are determined by the roots of the transcendental equation

$$-q+q^2\varphi(s)\psi(q)=0 \qquad (2.5)$$

There are two roots in the case of the upper branch $\psi(q)$ from (1.2): a zero root and non-zero on $q^* \neq 0$. Equation (2.5), which has been linearized close to $q = 0$, has a growing solution $q = \exp T$, and this root is therefore unstable. The solution of the equation which has been linearized close to $q = q^*$ decreases as $q = \exp(-\alpha T)$, $\alpha = 1 + q^*\psi'(q^*)/\psi(q^*) > 0$, and the root q^* is therefore stable. There are no steady-state solutions on the lower branch. Any solution $q = q(T)$ (apart from $q \equiv 0$) tends to a stable equilibrium position $q^* = q^*(s)$. In this case, the motion $q(T)$ to q^* occurs along the upper branch (Fig. 1) in a finite time.

The stability conditions are obtained under the assumption that $\varphi(s) = \text{const}$. However, it is necessary to take account of the fact that the saturation $s \approx s^0(x, t)$ depends on the "slow" time and increases monotonically. Correspondingly, the function $\varphi(s)$ also changes and the point of equilibrium therefore moves such that $q^* = q^*(s(x, t))$ increases monotonically with respect to t. For this reason, the quantity $q \cong q^0(T, x, t)$ crosses the point q^* after a finite time. At the instant when crossing occurs, there is a breakdown of the system onto the lower branch in accordance with (2.2). On this branch, the values of the pressure gradient decrease to a magnitude C_0, at which value there is a transfer onto the upper branch and the process is repeated.

The two-valued property of the function $\psi(q)$ (hysteresis) and the slow changes in the function $s(x, t)$ are the cause of the rapid oscillations. They also determine the character of the fast oscillations, in particular, the amplitude and period. It should be noted that the amplitude and period vary slowly due to the motion of the point of breakdown (Fig. 2).

The period of the oscillations on the slow time scale is estimated from the order of magnitude of $O(\tau_1\ln(1/\tau_1))$. This estimate is obtained from a calculation of the time of the motion from one breakdown point $q_0^*(x, t_0)$ to another $q_1^*(x, t_1)$. Bearing in mind that the velocity of the motion of the point $q^*(x, t)$ is finite, the time of the motion on the slow scale is estimated from the distance $t^* = t_1 - t_0 = O(|q_0^* - q_1^*|)$. The time of motion of the current point $q(T, x, t)$ in this interval on the fast time scale is determined during the integration of the first equation of system (2.2) for the leading term of expansion (2.3).

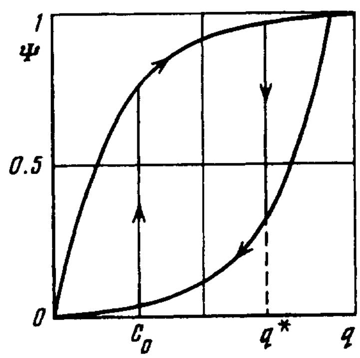

Fig. 1.

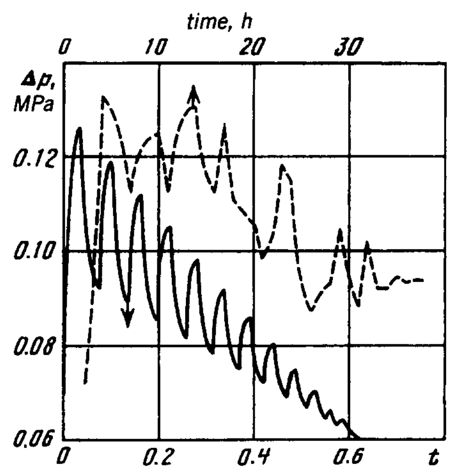

Fig. 2.

$$T^* = \int_{q_0^*}^{q_1^*} \frac{dq}{q - q^2\varphi\psi(q)}, \quad \varphi = \varphi(s(t_0))$$

The lower branch of the function $\psi(q)$ is chosen when $q \in [C_0, q_0^*]$ and the upper branch when $q \in [C_0, q_1^*]$. Since a $q_0^*(x, t_0)$ is first-order zero of the integrand of the function in the upper branch, then, for $q_1^*(x, t_1)$ close to $q_0^*(x, t_0)$, the estimate $T^* = O(|\ln|q_0^* - q_1^*||)$ holds. Taking account of the relation between the fast and slow times, we obtain the relation $t^* = \tau_1 O(|\ln t^*|)$, whence, in the first approximation, $t^* = \tau_1 O(|\ln \tau_1|)$.

The conclusions of the asymptotic analysis are confirmed by the results of numerical calculations using model (1.1)–(1.4).

3. NUMERICAL ANALYSIS OF THE SYSTEM

The system of equations (1.1)–(1.4) was solved numerically [5] in order to investigate the qualitative features of the proposed model of two-phase seepage. Equations (1.1) and the second equation of (1.3) were approximated by special finite-difference schemes with an exponential adjustment which take account of the existence of small parameters accompanying the higher derivatives. [6] Equations (1.4) and (1.7) were integrated numerically using the well-known "implicit" pressure–"explicit" saturation scheme [5]. In order to ensure the stability and the necessary accuracy of the difference schemes a spatial step of the variable $h = 0.01$ and a time step $l = 0.0001$ were chosen. The pressure drop Δp as a function of the dimensionless time t, which is equal to the ratio of the volume of the fluid pumped into the pore volume, is represented by the solid curve in Fig. 2. The calculations were carried out for

$$\mu_0 = 0.2, \quad \tau_1 = 0.01, \quad \tau_2 = 0.01$$

$$k_1(s) = 0.138\left(\frac{s - s_c}{s_T - s_c}\right)^{5/2}, \quad k_2(s) = 0.505\left(\frac{s_T - s}{s_T - s_c}\right)^{3/2}$$

It can be seen that pressure drop oscillations arise in a system with non-linear properties. The unit of dimensionless time corresponds to 2×10^4 s. The elastic properties of the stratum system amplify the pulsation in Δp, which are observed until the saturation throughout the whole length of the model attains a limit value s_T. When $\tau_1 = 0$, there are no oscillations in Δp. This means that the existence of a lag in the structural rearrangement processes of rheologically complex media leads to their occurrence.

The use of a seepage relaxation law (the second equation of (1.3)) leads to a retardation of the displacement process. Saturation profiles, calculated when $t_2 \neq 0$, lag behind the profile when $\tau_2 = 0$, while the structure of the saturation front barely changes. Calculations were carried out for various values of τ_1, τ_2, μ_0, k, V_0. Variation of these quantities leads to a change in the amplitude and frequency of the oscillations.

4. CONCLUSION

Hence, it has been shown that oscillations of the pressure drop with time arise during two-phase seepage in a porous medium and these oscillations are the result of the combined effect of the non-linear properties of a microemulsion and the effects of a time lag in the establishment of the phase permeabilities in immiscible displacement processes.

The theoretically observed pressure drop oscillations are qualitatively confirmed by the data from laboratory investigations into the seepage characteristics of porous media. The experimental dependence of the pressure drop on time is shown by the dashed curve in Fig. 2. This experimental dependence was obtained for the displacement of oil by water from a model BS_{1-5} stratum under the conditions for the Prirazlomnyi bed ($L = 0.1647$ m, $m = 0.17$, $k = 0.021\ \mu m^2$, $F_2 = 0.58$, $s_c = 0.285$, $s_T = 0.718$, $V_0 = 232.32$ m/year and $\mu_0 = 0.27$).

The model described enables one to interpret the pressure gradient oscillations observed in experiments on the displacement of oil by water from a sample of a porous medium.

It can therefore be concluded that pressure drop oscillations, which are usually considered as unfortunate, interference and experimental errors, can serve as a source of useful information in determining of the collecting and seepage characteristics of a stratum.

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